

(3 Hours)

Total Marks: 80

- N.B.** : (1) Question No. 1 is compulsory.
 (2) Attempt any three questions out of remaining five questions

- Q.1 (a) Investigate for what value of λ and μ the equations (5)
 $2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$
 has (i) No solution (ii) Unique solution (iii) Infinite number of solutions.
 (b) Draw a cumulative frequency curve “less than” for the following data. (5)

Monthly Income in ₹	No. of families
15000-17000	70
17000-19000	100
19000-21000	120
21000-23000	150
23000-25000	100
25000-27000	60

- (c) Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} x + cy^2, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

 (i) Find the constant c
 (ii) Find $P(0 \leq X \leq \frac{1}{2}, 0 \leq Y \leq \frac{1}{2})$
 (d) Find the stationary points of the function (5)
 $Z = 2x_1 + x_3 + 3x_2x_3 - x_1^2 - 3x_2^2 - 3x_3^2 + 17$

- Q.2 (a) Find the dimension and basis for the four fundamental subspaces for (10)

$$A = \begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$$

- (b) Let X and Y be jointly normal random variables with parameters (10)
 $\mu_x = 2, \sigma_x^2 = 1, \mu_y = 0, \sigma_y^2 = 4, r = \frac{1}{2}$
 i. Find $P(3X + 2Y \leq 7)$
 ii. Find $\text{Cov}(X + Y, X - Y)$

- Q.3 (a) Two samples were drawn from two populations and the results were as given below: (10)

Population	A	B
Sample Size	19	13
Sample S.D	50	35

Test the hypothesis that variance of A is greater than or equal to variance of B. (for degrees of freedom 12 and 18 at 5% $F = 2.34$).

- (b) Represent the following data by a subdivided bar diagram. The data represents figures of production of paper in thousand tonnes for the years 2015, 2016, 2017. Also represent the data by a percentage bar diagram. (10)

Types	2015	2016	2017
Printing and Writing	35	40	50
Wrapping	18	19	15
Boards	16	15	15
Special Varieties	7	5	5
Total	76	79	85

- Q.4 (a) Find 3 yearly moving averages and represent these on a graph paper. Also represent the original time series on the graph. (10)

Year	1999	2000	2001	2002	2003	2004	2005	2006	2007
Sales (in lakhs)	51	53	50	57	60	55	59	62	68

- (b) Minimize $f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 4)^2$ Subject to $h(x_1, x_2) = x_1 + x_2 - 5 = 0$, using parabolic Penalty method. (10)

- Q.5 (a) Minimize $f(x_1, x_2) = x_1^2 - x_1x_2 + x_2^2$ starting from the point $X_1 = (1, 1/2)$ using steepest descent method. (10)

- (b) Explain the need for exploratory data analysis. Also list and explain exploratory data analysis techniques. (10)

- Q.6 (a) Describe with example and action to be taken for the following (10)

1. Data Cleaning
2. Irrelevant data
3. Incorrect data
4. Data cleaning
5. Outliers

- (b) Write a short note on linear discriminant analysis techniques and principal component analysis algorithm (10)